SAMPLE PAPER 4: PAPER 1

QUESTION 6 (25 MARKS)

Question 6 (a)

$f(x)$ is continuous as there are no gaps.

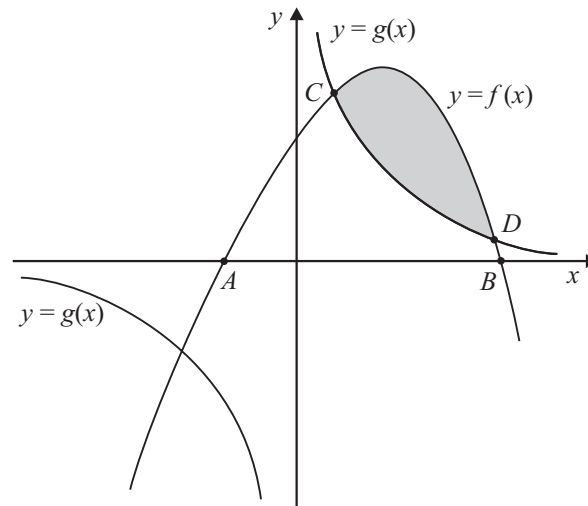$g(x)$ is not continuous at there is a gap at $x = 0$.

Question 6 (b)

$y = f(x) = -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3}$

$f'(x) = -\frac{7}{3}x + \frac{17}{6}$

$f'(x) = 0 \Rightarrow -\frac{7}{3}x + \frac{17}{6} = 0$

$-14x + 17 = 0$

$\therefore x = \frac{17}{14}$

Question 6 (c)

$y = f(x) = -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3}$

$y = 0 \Rightarrow -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3} = 0$

$7x^2 - 17x - 62 = 0$

$(7x - 31)(x + 2) = 0$

$x = -2, \frac{31}{7}$

$\therefore A(-2, 0),\ B(\frac{31}{7}, 0)$

Question 6 (d)

$(1, 12) \in f(x)?$

$f(x) = -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3}$

$f(1) = -\frac{7}{6}(1)^2 + \frac{17}{6}(1) + \frac{31}{3}$

$\quad = -\frac{7}{6} + \frac{17}{6} + \frac{31}{3} = \frac{36}{3} = 12$ (True)

$(4, 3) \in f(x)?$

$f(x) = -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3}$

$f(4) = -\frac{7}{6}(4)^2 + \frac{17}{6}(4) + \frac{31}{3}$

$\quad = -\frac{56}{3} + \frac{34}{3} + \frac{31}{3} = \frac{9}{3} = 3$ (True)

$(1, 12) \in g(x)?$

$g(x) = \dfrac{12}{x}$

$g(1) = \dfrac{12}{1} = 12$ (True)

$(4, 3) \in g(x)?$

$g(x) = \dfrac{12}{x}$

$g(4) = \dfrac{12}{4} = 3$ (True)

Question 6 (e)

$A = \displaystyle\int_1^4 (f(x) - g(x))\,dx$

$= \displaystyle\int_1^4 \left(-\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3} - \frac{12}{x} \right) dx$

$= [-\frac{7}{18}x^3 + \frac{17}{12}x^2 + \frac{31}{3}x - 12\ln x]_1^4$

$= \{-\frac{7}{18}(4)^3 + \frac{17}{12}(4)^2 + \frac{31}{3}(4) - 12\ln(4)\} - \{-\frac{7}{18}(1)^3 + \frac{17}{12}(1)^2 + \frac{31}{3}(1) - 12\ln(1)\}$

$= \frac{111}{4} - 12\ln 4 \approx 11.1$